**Are you the fittest gym rat?**

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[**Are You The Fittest Gym Rat? | FiveThirtyEight**](https://fivethirtyeight.com/features/are-you-the-fittest-gym-rat/)

This problem is much simpler in the case where there are many gym members. I set up a system where every member corresponds to number ranging from 0 to N-1, where the higher attending members have higher numbers. The probability a given member is at the gym at a given time is then:

For large N, this is simply x/N:

Then I calculate the expected number of members at the gym at a given time:

The probability that a given member will be the first person I see is the probability that member is at the gym (x/N) times the probability I will see them first (1/E = 2/N). I add the probabilities for members N/2 to N who have higher probabilities than me of being at the gym (i.e. greater than ½).

As a sanity check, notice that the integral over the range 0 to N equals 1, as it must since somebody must be the first person I see.

To find the probability for smaller numbers of members, I must consider two cases: Even number of members and odd number of members (not including myself). For both cases, the expected number of members present at any given time is still N/2.

For even N, the attendance rate of the N/2 member is greater than 50% (N/2/(N-1)>50%). So the probability the first member I see attends more than me is:

For N=2 to 10, the probabilities are:

I searched for the sequence 2,10,24,44,70 in the Online Encyclopedia of Integer Sequences and found it here: [A049450 - OEIS](https://oeis.org/A049450) , “Pentagonal numbers multiplied by 2: a(n) = n\*(3\*n-1).” “n” in this formula is “N/2.”

Inserting this formula into the equation for the probability, I get:

For odd N, attendance percentage for member N/2-1/2 is 50%.

So the first member who has greater attendance than me is member N/2+1/2. The probability the first member I see has greater attendance than me is given by this sum:

For N=3 to 11, the probabilities are:

Again I searched the OEIS, this time for sequence 4, 14, 30, 52, 80. I found this result: [A049451 - OEIS](https://oeis.org/A049451) “Twice second pentagonal number.” This sequence has formula: a(n) = n\*(3\*n+1), where n=N/2-1/2.

Inserting this formula into the equation for the probability, I get:

Here’s a graph showing the probability jumping between (3N-2)/(4N-4) for even N and (3N-1)/4N for odd N. As expected, the probability converges to 75% as N increases.

